

The Effect of Extra Dimension on Dark Matter ^a

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ABSTRACT

We investigate the thermal relic density of a cold dark matter in the brane world cosmology. Since the expansion law in a high energy regime is modified from the one in the standard cosmology, if the dark matter decouples in such a high energy regime its relic number density is affected by this modified expansion law. We derive analytic formulas for the number density of the dark matter. It is found that the resultant relic density is characterized by the “transition temperature” at which the modified expansion law in the brane world cosmology is connecting with the standard one, and can be considerably enhanced compared to that in the standard cosmology, if the transition temperature is low enough. The implication to the neutralino dark matter also is mentioned.

1. Introduction

Recent various cosmological observations have established the Λ CDM cosmological model with a great accuracy, where the energy density in the present universe consists of about 73% of the cosmological constant (dark energy), 23% of non-baryonic cold dark matter and just 4% of baryons. However, to clarify the identity of the dark matter particle is still a prime open problem in cosmology and particle physics.

In the case that the dark matter is the thermal relic, we can estimate its number density by solving the Boltzmann equation

$$\frac{dn}{dt} + 3Hn = -\langle\sigma v\rangle(n^2 - n_{EQ}^2), \quad (1)$$

with the Friedmann equation,

$$H^2 = \frac{8\pi G}{3}\rho, \quad (2)$$

where H is the Hubble parameter, n is the actual number density of dark matter particles, n_{EQ} is the number density in thermal equilibrium, $\langle\sigma v\rangle$ is the thermal averaged product of the annihilation cross section σ and the relative velocity v , ρ is the energy density, and G is the Newton's gravitational constant. Eq. (1) is rewritten as

$$\frac{dY}{dx} = -\frac{s\langle\sigma v\rangle}{xH}(Y^2 - Y_{EQ}^2), \quad (3)$$

^aPresented by O. Seto.

in terms of the number density to entropy ratio $Y = n/s$ and $x = m/T$. As is well known, the final number density of dark matter particles to entropy ratio is given as

$$Y(\infty) \simeq \frac{x_d}{\lambda \left(\sigma_0 + \frac{1}{2} \sigma_1 x_d^{-1} \right)}, \quad (4)$$

with a constant

$$\lambda = \frac{xs}{H} = 0.26 \left(\frac{g_{*S}}{g_*^{1/2}} \right) M_P m \quad (5)$$

for models in which $\langle \sigma v \rangle$ is approximately parametrized as $\langle \sigma v \rangle = \sigma_0 + \sigma_1 x^{-1} + \mathcal{O}(x^{-2})$, where g_* is the effective total number of relativistic degrees of freedom, $x_d = m/T_d$, T_d is the decoupling temperature and m is the mass of the dark matter particle, and $M_P \simeq 1.2 \times 10^{19} \text{ GeV}$ is the Planck mass [1].

Recently, the brane world models have been attracting a lot of attention as a novel higher dimensional theory. In these models, it is assumed that the standard model particles are confined on a “3-brane” while gravity resides in the whole higher dimensional spacetime. The model first proposed by Randall and Sundrum (RS) [2], the so-called RS II model, is a simple and interesting one, and its cosmological evolution have been intensively investigated [3]. In the model, our 4-dimensional universe is realized on the 3-brane with a positive tension located at the ultra-violet boundary of a five dimensional Anti de-Sitter spacetime. The Friedmann equation for a spatially flat spacetime in the RS brane cosmology is found to be

$$H^2 = \frac{8\pi G}{3} \rho \left(1 + \frac{\rho}{\rho_0} \right), \quad (6)$$

where $\rho_0 = 96\pi G M_5^6$ with M_5 being the five dimensional Planck mass, and we have omitted the four dimensional cosmological constant and the so-called dark radiation. The second term proportional to ρ^2 is a new ingredient in the brane world cosmology and lead to a non-standard expansion law.

Here, we investigate the brane cosmological effect for the relic density of the dark matter due to this non-standard expansion law. If the new terms in Eq. (6) dominates over the term in the standard cosmology at the freeze out time of the dark matter, they can cause a considerable modification for the relic abundance of the dark matter [4].

2. Relic density in brane cosmology

We are interested in the early stage in the brane world cosmology where the ρ^2 term dominates, namely, $\rho^2/\rho_0 \gg \rho$. In this period, the coupling factor of collision term in the Boltzmann equation is given by

$$\frac{s\langle \sigma v \rangle}{xH} \simeq \frac{\lambda}{x_t^2} \langle \sigma v \rangle \quad (7)$$

where a new temperature independent parameter x_t is defined as

$$x_t^4 \equiv \left. \frac{\rho}{\rho_0} \right|_{T=m}. \quad (8)$$

Note that the evolution of the universe can be divided into two eras. At the era $x \ll x_t$ the ρ^2 term in Eq. (6) dominates (brane world cosmology era), while at the era $x \gg x_t$ the expansion law obeys the standard cosmological law (standard cosmology era). In the following, we call the temperature defined as $T_t = mx_t^{-1}$ (or x_t itself) “transition temperature” at which the evolution of the universe changes from the brane world cosmology era to the standard cosmology era. We consider the case that the decoupling temperature of the dark matter particle is higher than the transition temperature.

At the early time, the dark matter particle is in the thermal equilibrium and $Y = Y_{EQ} + \Delta$ tracks Y_{EQ} closely. After the temperature decreases, the decoupling occurs at x_d roughly evaluated as $\Delta(x_d) \simeq Y(x_d) \simeq Y_{EQ}(x_d)$. The solutions of the Boltzmann equation during the $x_t > x > x_d$ epoch are given as

$$\frac{1}{\Delta(x)} - \frac{1}{\Delta(x_d)} = \begin{aligned} & \frac{\lambda\sigma_0}{x_t^2} (x - x_d), \\ & \frac{\lambda\sigma_1}{x_t^2} \ln\left(\frac{x}{x_d}\right), \\ & \frac{\lambda\sigma_n}{x_t^2} \left(\frac{1}{n-1}\right) \left(\frac{1}{x_d^{n-1}} - \frac{1}{x^{n-1}}\right), \end{aligned} \quad (9)$$

for $n = 0$ (S-wave process), $n = 1$ (P-wave process), and $n > 1$ respectively. Here we have parametrized $\langle\sigma v\rangle$ as $\langle\sigma v\rangle = \sigma_n x^{-n}$. Note that $\Delta(x)^{-1}$ is continuously growing without saturation for $n \leq 1$. This is a very characteristic behavior of the brane world cosmology, comparing the case in the standard cosmology where $\Delta(x)$ saturates after decoupling and the resultant relic density is roughly given by $Y(\infty) \simeq Y(x_d)$. For a large $x \gg x_d$ in Eq. (9), $\Delta(x_d)$ and x_d can be neglected. When x becomes large further and reaches x_t , the expansion law changes into the standard one, and then Y obeys the Boltzmann equation with the standard expansion law for $x \geq x_t$. Since the transition temperature is smaller than the decoupling temperature in the standard cosmology (which case we are interested in), we can expect that the number density freezes out as soon as the expansion law changes into the standard one. Therefore the resultant relic density can be roughly evaluated as $Y(\infty) \simeq \Delta(x_t)$ in Eq. (9).

Now, we show analytic formulas of the final relic density of the dark matter in the brane world cosmology. For $n = 0$, we find the resultant relic density

$$Y(\infty) \simeq 0.54 \frac{x_t}{\lambda\sigma_0}. \quad (10)$$

in the case of $x_d \ll x_t$. Note that the density is characterized by the transition temperature x_t as we expected. By using the well known formula (4), for a given $\langle\sigma v\rangle$, we obtain the ratio of the energy density of the dark matter in the brane world cosmology ($\Omega_{(b)}$) to the one in the standard cosmology ($\Omega_{(s)}$) such that

$$\frac{\Omega_{(b)}}{\Omega_{(s)}} \simeq 0.54 \left(\frac{x_t}{x_{d(s)}}\right), \quad (11)$$

where $x_{d(s)}$ is the decoupling temperature in the standard cosmology. Similarly, for $n = 1$ we find the resultant relic density

$$Y(\infty) \simeq \frac{x_t^2}{\lambda \sigma_0 \ln x_t} \quad (12)$$

in the case of $x_d \ll x_t$. Then, the ratio of the energy density of the dark matter is found to be

$$\frac{\Omega_{(b)}}{\Omega_{(s)}} \simeq \frac{1}{2 \ln x_t} \left(\frac{x_t}{x_{d(s)}} \right)^2. \quad (13)$$

We can obtain results for the case of $n > 1$ in the same manner.

3. Summary

We have investigated the thermal relic density of the cold dark matter in the brane world cosmology. If the five dimensional Planck mass is small enough, the ρ^2 term in the modified Friedmann equation can be effective when the dark matter is decoupling. We have derived the analytic formulas for the relic density and found that the resultant relic density can be enhanced. The enhancement factor is characterized by the transition temperature, at which the evolution of the universe changes from the brane world cosmology era to the standard cosmology era.

If our scenario is applied to the supersymmetric dark matter such as the neutralino, its abundance could be enhanced. Thus a larger annihilation cross section, as in the higgsino-like case, may be favorable if the ρ^2 term was significant at the decoupling time. Allowed regions obtained in the previous analysis in the standard cosmology [5] would be dramatically modified in such cases.

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